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Inventory control

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- Inventory is stock on hand and it is kept to meet unforseen demand or unexpected demands. Inventory can be in the form of raw material, semi finished goods or in the form of final product.

- ## Inventory control :

  Inventory control is concerned with achieving an optimum balance between two competing objectives. The objectives are :

  (i) to minimize investment in industry,

  (ii) to maximize the service levels to the firm's customers. and own operating departments.

- ## Inventory classification :

  Inventory may be classified as follows :

  (i) Raw inventories → They include raw material and semifinished products supplied by another firm and which are raw items for the present industry.

  (ii) In-process inventory → They are semi-finished goods at various stages of manufacturing cycle.

  (iii) Finished inventories → They are finished goods lying in stock rooms and waiting dispatch.

  (iv) Indirect inventories → They include lubricants and other items (like spare parts) needed for proper operation, repair and maintenance during manufacturing cycle.

Inventory management :

To manage these various kinds of inventories two alternative control procedures can be used.

(1.) Order point system :

- It is a traditional approach to inventory control. In this system the items are restocked when the inventory levels become low.
- Lot size and re order point calculations are the more spectacular aspect of inventory management.

(2.) Materials requirement planning (MRP) :

- MRP is the technique used to plan and control manufacturing inventories.
- It is a computational technique that converts the master schedule for end products into a detailed schedule for the raw material and components used in the end products.
- In general, MRP is an appropriate control procedure for type ① and ⑪ inventories.
- Order point systems are often considered as the appropriate procedure to control inventory types ⑪ and ⑩.

Types of cost

- Purchase cost
- ordering cost (or) Set up cost
- Holding cost (or) Carrying cost
- Shortage cost

- **Purchase cost** = (Annual demand) x (cost per unit)

$$\boxed{P.C. = D \times C}$$, D = annual demand
c = cost per unit (Rs/unit)

- **Ordering cost** – cost associated with chasing an order. It includes communication, transportation etc.

ordering cost = (No. of orders) x (cost per order)

$$\boxed{O.C. = N \times C_0} \quad , \quad \boxed{O.C. = C_0 \times \frac{D}{Q}}$$

Annual demand (D).

let D = 6000 units
Q = 200 units → Quantity placed per order.

$\therefore$ Quantity placed per order (N) = $\frac{6000}{200} = 30.$

$$\boxed{No.\ of\ order = \frac{D}{Q}}$$

- **Set up cost**

It includes internal production system. Items are manufactured within the production system. It includes cost associated with bringing a shut down system into starting position. It also includes maintenance cost.

$$\boxed{Set\ up\ cost = No.\ of\ sets\ ups \times cost\ per\ setup}$$

Holding cost : It is the cost associated with storage of items in inventory taken over per unit year.

Q = 1000 units

C_h = holding cost / unit / year

eg 2 rupees/unit/year,

Holding cost = (Average inventory) x (Holding cost per unit per year)

$$\boxed{H.C = \frac{Q}{2} \times C_h} \quad = \frac{1000}{2} \times 2 = \underline{1000 \text{ rupees}}$$

↳ H.C

if C_h = 0.5 rupees/unit/month

↳ but it should be per year.

∴ C_h = 0.5 x 12 = 6 rupees/unit/year

$\boxed{C_h = 10\%}$ if c = 500 rupees/unit

and C_h = 10% of unit cost

then C_h = 50 rupees/unit/year.

Shortage cost or Back order cost : It is the cost associated with storage of units. It includes customer good will loss, the loss associated with no sale of items.

Shortage cost = (Average shortage) x (Back order cost)/shortage

$$\boxed{S.C. = \frac{S}{2} \times C_b}$$

C_b = cost associated every time the unit goes out of stock.

Total annual cost (TAC) = P.C. + O.C. + H.C.

$$\boxed{T.A.C. = D\times C + \frac{D}{Q}\times C_o + \frac{Q}{2}\times C_h}$$

$$TAC = P.C. + \underbrace{TIC}$$

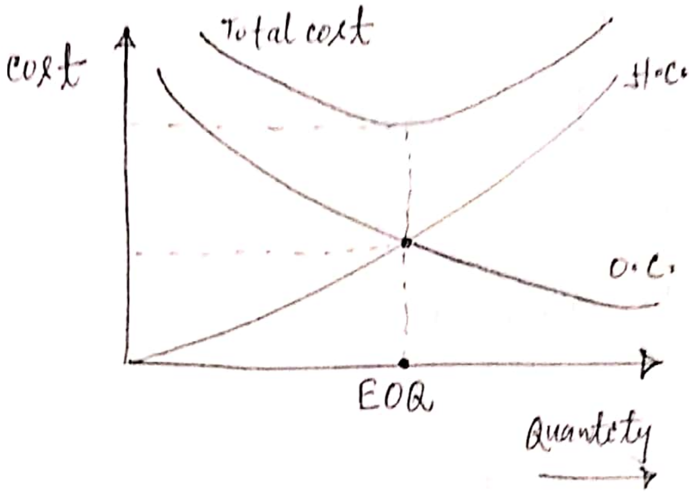
↳ Total inventory cost

Total inventory cost (TIC) = O.C. + H.C.

$$\boxed{TIC = \frac{D}{Q}C_o + \frac{Q}{2}C_h}$$

if Q ↑, O.C. ↓

and if Q ↑, H.C. ↑



Analytical method

$$T.I.C. = \frac{D}{Q}C_o + \frac{Q}{2}C_h$$

$$\frac{d(T.I.C.)}{dQ} = 0 \Rightarrow \frac{d(TIC)}{dQ} = \frac{-D}{Q^2}C_o + \frac{C_h}{2} = 0$$

$$\Rightarrow \frac{C_h}{2} = \frac{D}{Q^2}C_o$$

$$\Rightarrow Q^2 = \frac{2DC_o}{C_h}$$

$$\Rightarrow \boxed{Q^* = \sqrt{\frac{2DC_o}{C_h}}}$$

Graphical method

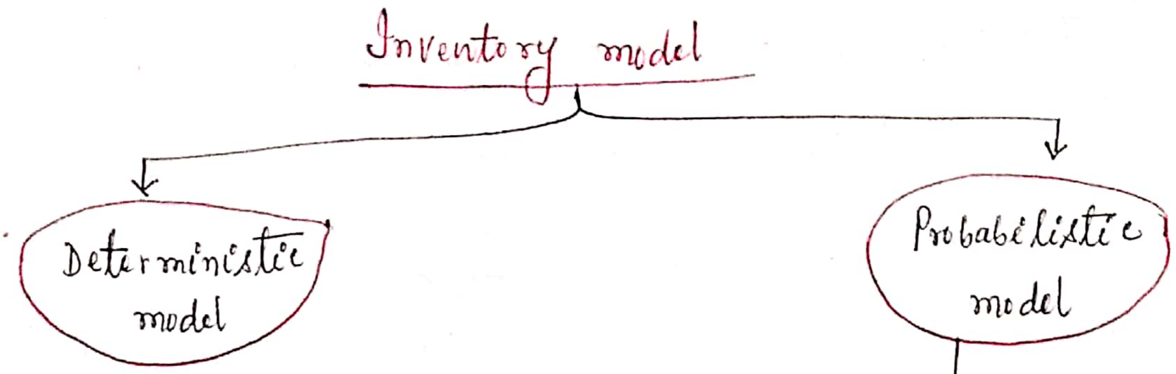
if O.C. = H.C. {∵ TIC is minimum}

$$\frac{D}{Q}C_o = \frac{Q}{2}C_h$$

$$Q^2 = 2D\frac{C_o}{C_h}$$

$$\boxed{Q^* = \sqrt{\frac{2DC_o}{C_h}}}$$

S.



- Lead time and demand rate is uniform through out the year.

1. EOQ model
   (Economic order quantity model)
2. Discount model (or) Price-Break model
3. Production - Consumption model
4. Shortage model

Demand rate and lead time is not uniform.

* E.O.Q. Model :-

T.I.C. minimum at $Q^* = \sqrt{\frac{2DC_0}{C_h}}$

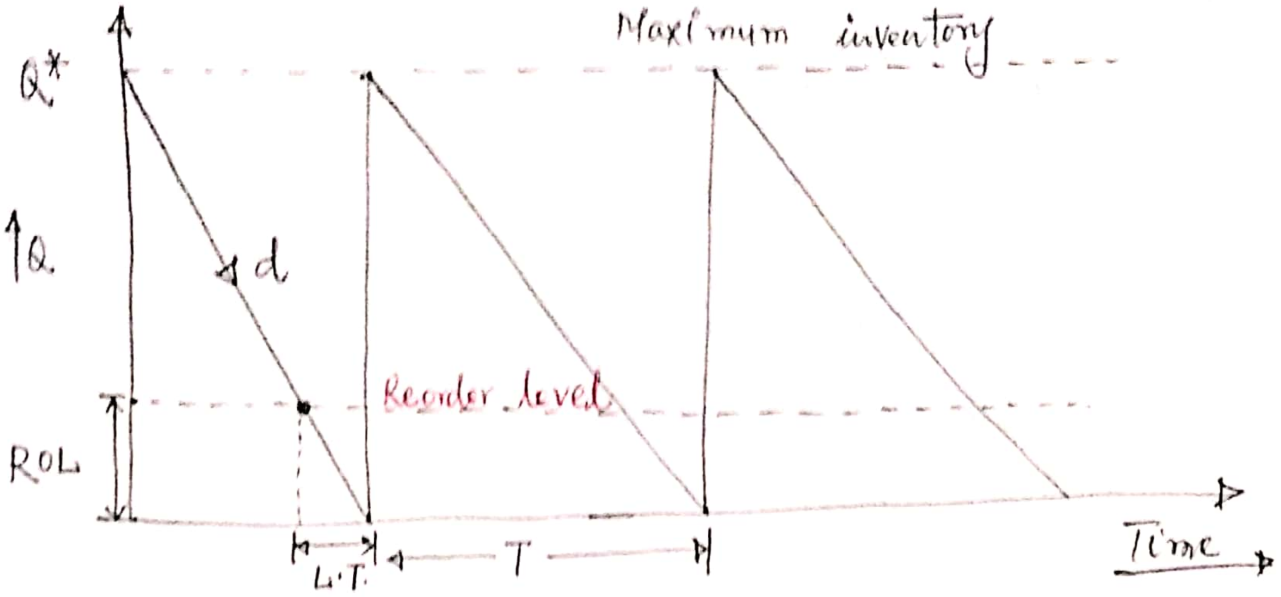
Let us calculate the minimum total inventory cost

$$T.I.C. = H.C. + O.C. \qquad \{\text{at EOQ} \ni H.C = OC\}$$

$$T.I.C.^* = 2 \times H.C.$$

$$= 2 \times \frac{Q^*}{2} C_h = Q^* \times C_h = \sqrt{\frac{2DC_0}{C_h}} \times C_h = \sqrt{2DC_0C_h}$$

$$\boxed{T.I.C^* = \sqrt{2DC_0C_h}}$$



Also known as Infinite Replenishment rate.

d = demand rate

$$\boxed{ROL = d \times LT}$$

eg. Monthly expenditure = 9000 unit

so, d = 300 unit/day

LT = 5 day

ROL = 5 × 300 = 1500 units

T = Total time in which maximum inventory is consumed.

L.T. = (lead time) = time in which ROL quantity is consumed.

→ It is the time period between placing an order and getting the previous inventory consumed.

If annual demand of a product is 2555 units and lead time is 8 days. Then reorder point in number of units is ___?___.

$$R.O.L. = \frac{2555}{365} \times 8 = \underline{56 \text{ units}} \text{ Ans}$$

Ex. The demand for a particular item is 10,000 units/year. The ordering cost is rupees 100/year. The carrying cost is Rs. 0.5/unit/year. Determine—

① EOQ

② No. of order per year (N)

(iii) Time between orders

(iv) Reorder point if lead time is one month.

(v) Total cost when the unit cost is Rs. 2.

Soln: Annual demand (D) = 10,000 units/year

ordering cost (Co) = Rs. 100/order

Carrying or holding cost (C_h) = Rs. 0.5/unit/year

(i) EOQ $= \sqrt{\frac{2DCo}{C_h}} = \sqrt{\frac{2 \times 10{,}000 \times 100}{0.5}}$ = 2000 units

(ii) No. of orders per year $= \frac{D}{EOQ} = \frac{10{,}000}{2000} = 5$

(iii) Time between order ⇒ 5 orders in one year

⇒ 1 order in $\frac{12\text{ month}}{5}$ ~~= 2.4 month~~

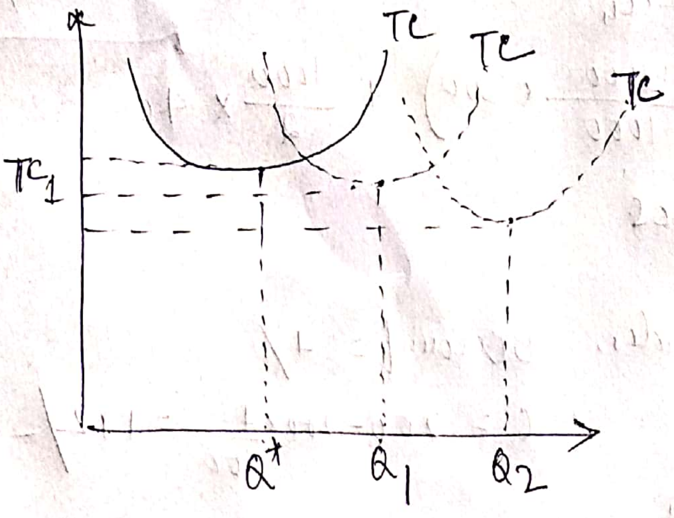
$\frac{365\text{ days}}{5}$ = 73 days

So, time between orders = 2.4 months

(iv) Reorder level = d × (LT)

$= \frac{10{,}000}{365} \times 30$ = 821.91 units

Model II Price Break (or) Discount Model



- Minimum cost be at a quantity order $Q > Q^*$.

- Best order size must be Calculated by considering Total annual cost which involves discount.

$$\boxed{TAC = PC + OC + HC}$$

Ex: Annual demand for window frame is 10,000. Each frame cost Rs. 200, order cost rupees 300/order, Inventory holding cost Rs. 40/frame/year. The supplier is willing to offer 2% discount if order quantity 1000 or more and 4% if order quantity is 2000 or more. If the total cost is to be minimized the retailer should —

ⓐ order 200 frame every time

ⓑ Accept 2% discount

ⓒ Accept 4% discount

(d) Economic order quantity (EOQ)

Soln:

$D = 10000.$

$C = 200$ rupees/frame

$C_o =$ Rs 300/order

$C_h =$ Rs. 40/frame/year

Now, $E.O.Q = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2 \times 10000 \times 300}{40}}$

$= 387.298$ Q^*

$T.I.C\Big|_{at\ Q^*} = \sqrt{2DC_oC_h} = 15491.93$ rupees

$\therefore$ Total annual cost at $Q^* = D \times C + T.I.C\Big|_{at\ Q^*}$

$= 10000 \times 200 + 15491.9$

$= \boxed{2015491.93}$ rupees

Now let Q_1 = 1000 quantity/order. , Discount = 2%

$C = 200 - 2$
$= 196/-$

$$T.A.C. = DC + \frac{D}{Q} C_o + \frac{Q}{2} \cdot C_h$$

$$= (10,000 \times \overset{196}{\cancel{200}}) + \left(\frac{10,000}{1000} \times 300\right) + \frac{1000}{2} \times 40$$

$$= \boxed{1983000} \text{ rupees}$$

Again, let Q_2 = 2000 quantities/order, Discount = 4%

$$\therefore C = 200 - 200 \times \frac{4}{100} = 192/-$$

$$\therefore T.A.C. = DC + \frac{D}{Q} C_o + \frac{Q}{2} C_h$$

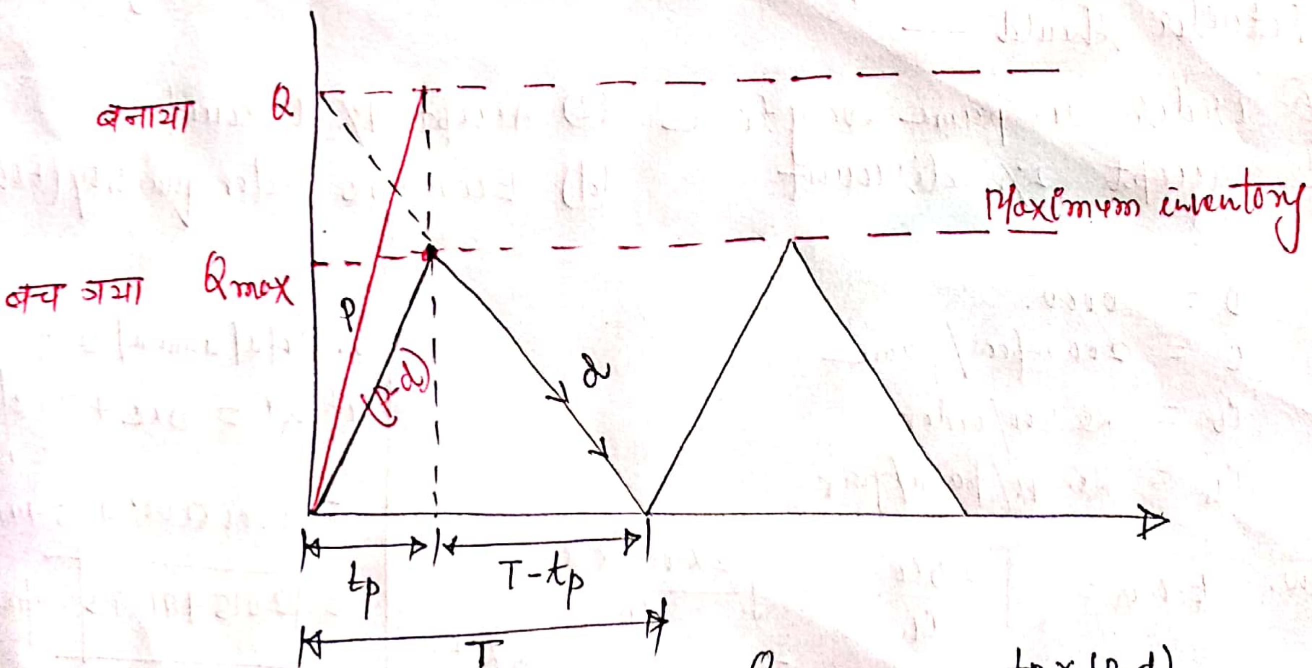
$$= (10,000 \times 192) \times \left(\frac{10,000}{2000} \times 300\right) + \left(\frac{2000}{2} \times 40\right)$$

$$= \boxed{1961500} \text{ rupees}$$

↳ It is the least T.A.C.

So, Best order size = 2000 Ans

* Model - III Production - Consumption model :



$Q = t_p \times P$

$Q_{max} = t_p \times (P-d)$

$$\frac{Q_{max}}{Q} = \frac{t_p \times (P-d)}{t_p \times P}$$

$$\boxed{Q_{max} = \left(\frac{P-d}{P}\right) Q}$$

$P = 80$ units/day, $d = 20$ units/day, Lot size $= 600$ units

Maximum inventory level $= \underline{?}$

$$Q_m = Q \times \left(\frac{P-d}{P}\right) = 600 \times \left(\frac{80-20}{80}\right) = \cancel{600}^{150} \times \frac{\cancel{60}^{3}}{\cancel{80}_{4}} = \underline{750 \text{ units}}$$

In this model, production and consumption takes place simultaneously until maxm inventory is reached. Then inventory is allowed to reduce down to zero with a uniform consumption rate. The production takes place upto time 'tp' at a faster rate than the consumption.

$$\text{Holding cost} = \frac{Q_{max}}{2} \times C_h$$

$$= \frac{Q}{2}\left(\frac{P-d}{P}\right) \times C_h.$$

At E.O.Q. - H.C. = O.C.

$$\frac{Q}{2}\left(\frac{P-d}{P}\right) C_h = \frac{D}{Q} \times C_o$$

$$\Rightarrow Q^2 = \frac{2DC_o}{C_h} \cdot \frac{P}{(P-d)}$$

$$\Rightarrow \boxed{Q^* = \sqrt{\frac{2DC_o}{C_h}} \times \sqrt{\frac{P}{P-d}}}$$

↳ production factor > 1.

Total inventory cost -

At EOQ -

$$T.I.C^* = H.C + O.C$$
$$= H.C + H.C$$
$$= 2\, H.C$$
$$= \cancel{2} \cdot \frac{Q_{max}}{\cancel{2}} \times C_h$$

$$T.I.C.^* = Q^* \left(\frac{P-d}{P}\right) \times C_h$$

$$\therefore T.I.C^* = \sqrt{\frac{2DC_o}{C_h} \cdot \frac{P}{P-d}} \left(\frac{P-d}{P}\right) C_h$$

$$\boxed{T.I.C^* = \sqrt{2DC_oC_h} \cdot \sqrt{\frac{P-d}{P}}}$$

↳ < 1.
Preferred as compared to first model.

Ex: A company produces and sells 5000 special bearings annually. The set up cost per order is Rs. 900/setup and factory cost is Rs. 50 each. Carrying cost on average finished goods is 20% of unit price and production rate is 100 per day.

(i) How many bearings are to be produced per production run?

(ii) What is maximum inventory level?

Sol

$D = 5000$ quantities / year

$C_o =$ Rs. 900 / setup

$C =$ Rs. 50

$C_h = 20\%$ of unit price $= 0.2 \times 50 = 10$ rupees

$P = 100/\text{day} = 36500/\text{year}$

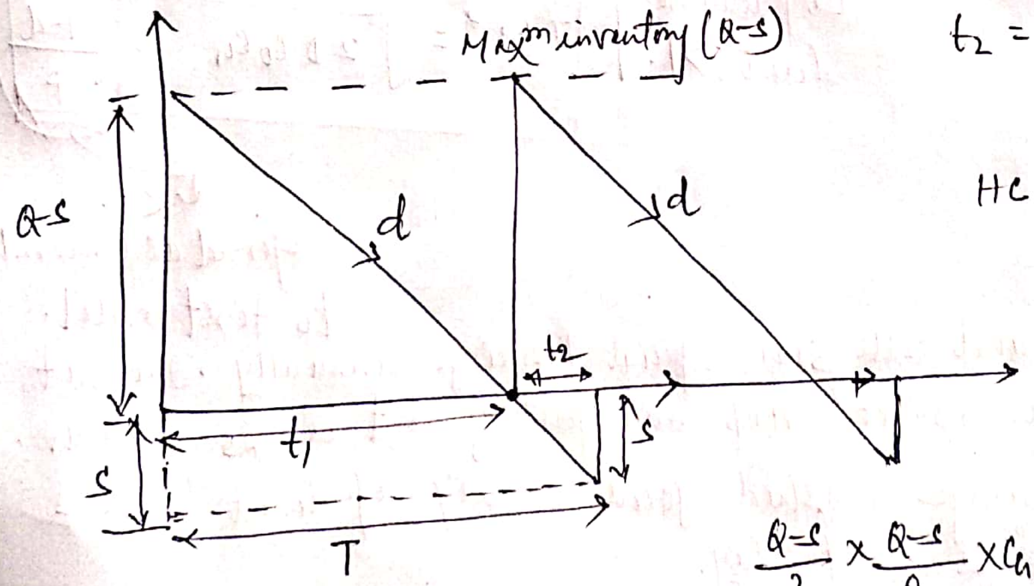
$d = 5000/\text{year}$.

(i) $\therefore \; Q^* = \sqrt{\frac{2DC_o}{C_h}} \cdot \sqrt{\frac{P}{P-d}} = \sqrt{\frac{2\times5000\times900}{10}} \times \sqrt{\frac{36500}{36500-5000}} \boxed{= 1021.20}$ ✓

(ii) $Q_{max} = Q^* \cdot \frac{P-d}{P} = 1021.20 \times \frac{36500-5000}{36500} = \underline{881.212}$

Model IV Shortage model.

In this model shortage will be allowed and it is very similar to the first model and only difference being that a planned back order ~~being~~ will be observed, where a customer places an order and finds the inventory out of stock.



t_1 = time without shortage

t_2 = time for which shortage is faced.

$HC = \left(\frac{Q-s}{2}\right) \cdot \frac{t_1}{T} \cdot C_h$

$Q = T \times d$

$Q-s = t_1 \times d$

$\frac{Q-s}{Q} = \frac{t_1}{T}$

$\frac{Q-s}{2} \times \frac{Q-s}{Q} \times C_h$

$\boxed{H.C = \frac{(Q-s)^2}{2Q} \times C_h}$

S = No. of units short or back order

C_b = Back order or shortest cost per unit back order per year (Rs./unit/year)

$$\boxed{T.I.C. = O.C + H.C + \underbrace{S.C}_{\text{shortage cost}}}$$

Shortage cost :

$$S.C = \frac{S}{2}\cdot\frac{t_2}{T}\cdot C_b$$

$S = t_2 \times d$ $\qquad \frac{S}{Q} = \frac{t_2}{T}$

$Q = T \times d$

$$S.C. = \frac{S}{2} \times \frac{S}{Q}\cdot C_b = \frac{S^2}{2Q}\cdot C_b$$

$$\therefore \boxed{TIC = \frac{D}{Q} C_o + \frac{(Q-S)^2}{2Q} C_h + \frac{S^2}{2Q}\cdot C_b}$$

At economic order quantity –

$$OC = (HC + SC)$$

$$\boxed{Q^* = \sqrt{\frac{2DC_o}{C_h}} \cdot \underbrace{\times\sqrt{\frac{C_b + C_h}{C_b}}}_{\text{cost factor} > 1}}$$

$$\boxed{T.I.C^* = \sqrt{2DC_oC_h} \cdot \underbrace{\sqrt{\frac{C_b}{C_b + C_h}}}_{CF < 1}}$$

* optimum number of units back ordered :– (S^*) :–

$$(Q^* - S^*) C_h = S^* \cdot C_b$$

$$\rightarrow \frac{Q^* - S^*}{S^*} = \frac{C_b}{C_h}$$

$$\rightarrow \frac{Q^*}{S^*} = 1 + \frac{C_b}{C_h}$$

$$\frac{Q^*}{S^*} = \frac{C_b + C_h}{C_h}$$

$$\boxed{S^* = Q^* \left(\frac{C_h}{C_b + C_h}\right)}$$

Now $M^* = Q^* - S^*$

$$= Q^* - Q^* \left(\frac{C_h}{C_b + C_h}\right)$$

$$\boxed{M^* = Q^* \cdot \frac{C_b}{C_b + C_h}}$$

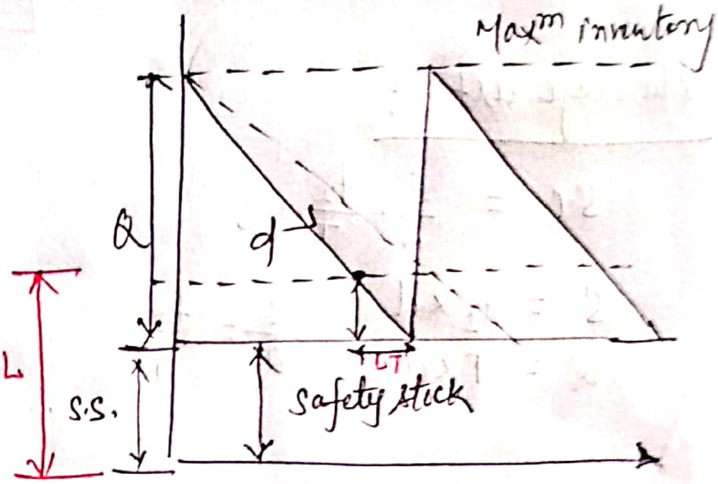
* Probabilistic Model:

In probabilistic model the demand rate or lead time may fluctuate and therefore the safety stock must be kept in order to provide better customer satisfaction.

Here, average inventory $= \frac{Q}{2} + SS$

Safety stock = Buffer stock

$$\boxed{ROL = (L.T \times d) + S.S.}$$



* Probability model → Profit-loss model
→ Service-level model.

(A) Profit loss model: This model is used for those items which get outdated very quickly, such as perishable items like fruits, flowers, vegitables etc. In order to maximize the profit we select an ordering quantity which is given as—

$$\boxed{P(s-1) < \frac{P}{P+L} \leq P(s)} \qquad \boxed{P(s-1) < \frac{C_s}{C_s+C_h} \leq P(s)}$$

P = Profit
L = loss
P(s) = Probability of 's' number to be sold.
↳ cumulative

A newspaper boy buys the paper of Rs. 2 and sells it in Rs. 2.4. If the daily demand of newspaper follows according to the distribution given below. How many papers he should ~~purchage~~ prochase to maximize his profit ?

X	P(X)	Cumulative Probability
30	0.05	0.05
31	0.15	0.20
32	0.4	0.60
33	0.25	0.85
34	0.15	1.00

} 0.167

loss = 2

~~gain~~ profit = 2.4 - 2 = 0.4

$$\therefore \frac{P}{P+L} = \frac{0.4}{0.4+2} = \frac{0.4}{2.4} = 0.167$$

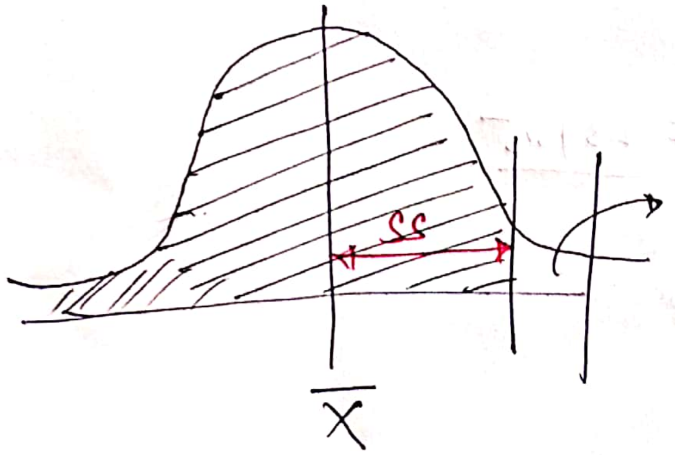
Here, 0.167 lies between 0.05 and 0.2, of these 0.2 is greater so he should purchase 31 papers daily to maximize his profit.

(B) **Service level model:** This model is associated to those condition where loss with respect to stock out is not known exactly. In this model, the level of safety stock is kept according to the service level the management want to achieve.

$$\text{Service level} = \frac{\text{Number of units supplied without delay}}{\text{No. of units demanded}} = \frac{95}{100} = 95\%$$

Let No. of units demanded = 100

" " " supplied without delay = 95



→ 5% chance of stockout

ss (safety stock) = 25

z = 1.645 for 95% service level

$ROL = \bar{x} + SS$

$$\boxed{ROL = \bar{x} + Z\sigma}$$

$Z\sigma$ = Safety stock

$\bar{x} = (LT \times d)$

Z	Service level
0	50%
0.84	80%
1.28	90%
1.645	95%
2.33	99%

* 95% service level is the standard value which means that 95% of the customer's demand is fulfilled during lead time and only 5% of customer's order are rejected because of stockout during lead time. The demand during lead time will be approximated by a normal distribution curve.

Q. For a product the unit cost is Rs 40. Average Weekend demand is 600 unit. The standard deviation for week is 150 units. Holding cost is Rs 0.1/unit/week and lead time is 4 weeks, then for 95% service level, find the safety stock and R.O.L.

Soln Safety stock (ss) $= Z.\sigma$, $\sigma = \sqrt{\sigma_1^2+\sigma_2^2+\sigma_3^2+\sigma_4^2}$

$= \sqrt{150^2+150^2+150^2+150^2}$

$= \sqrt{4 \times 150^2} = 300.$

$= 1.645 \times 300$

$= \boxed{493.5}$ ✓

$ROL = \bar{x} + SS$

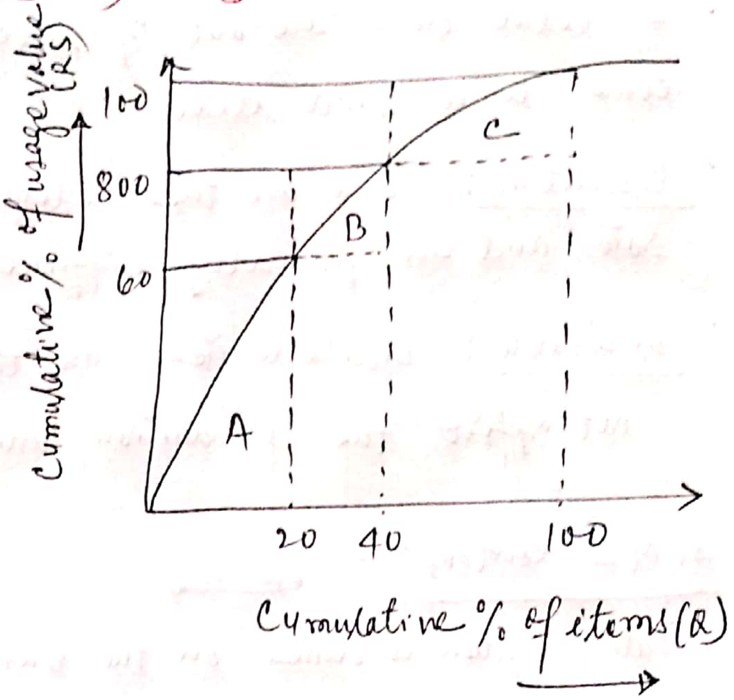
$= (LT \times d) + (SS)$

$= (4 \times 600) + 493.5 = \underline{2893.5}$

Inventory control

1. ABC Analysis [Pareto (80-20) rule]:

A – Always
B – Better
C – control



In this analysis, the inventory items in an organization are classified on the basis of their usage in monetary terms. It is very common to observe that usually a small number of items account for a large share of the total cost of materials and a comparatively large number of items involve an insignificant share. Based on this criterion, the items are divided into three categories as follows: –

A – high consumption value items
B – Moderate consumption value items
C – low consumption value item.

	item%	Usage%
Ⓐ	10–20%.	50–60%.
Ⓑ	30–40%.	30–40%.
Ⓒ	50–60%.	10–20%.

2. **VED** control — Vital essential, desirable [on the basis of importance]

- It is classified on the basis of importance.
- If vital items are out of stock then production unit will come to a stand still.
- **Essential:** These are those items which will affect the production rate and are generally difficult to obtain from market.
- **Desirable:** Desirable items are those which if not present, will not affect the production unit.

3. **HML control:—** ~~On the~~

- categorization is done on the basis of cost (unit cost).
- H → high cost, M → Medium cost, L → Low cost.

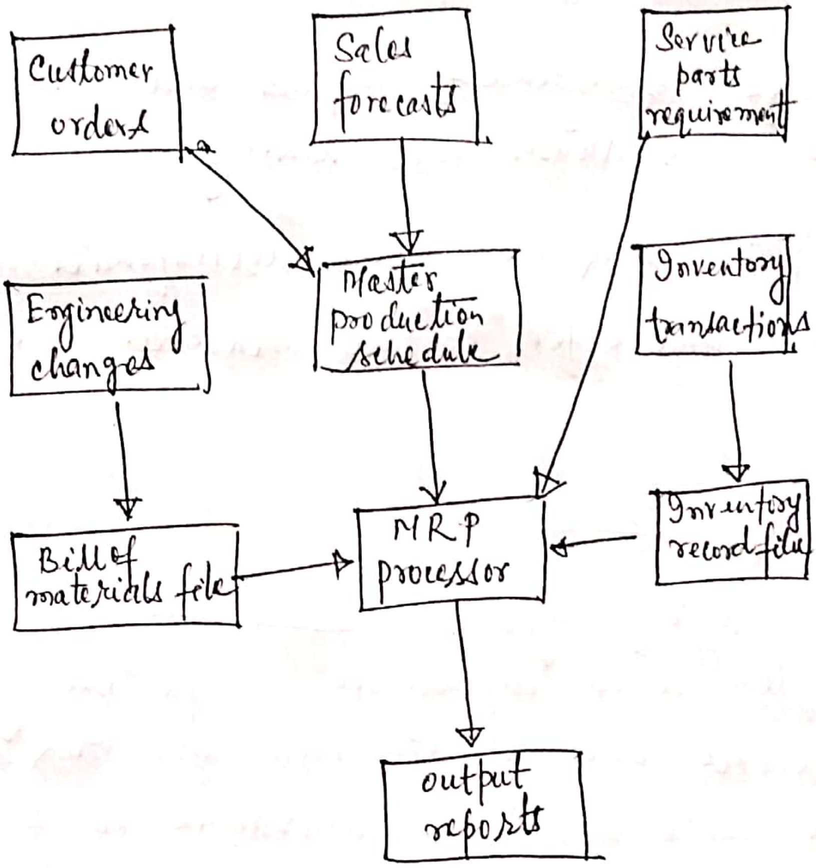
4. **SDE Control:—** (S → Scarce, D → Difficult, E → Easy)

- Inventories are classified according to the availability of items.

Material requirement Planning (MRP)

- MRP is a computational technique that converts the master schedule for end products into a detailed schedule for raw materials and components used in the end products.
- The detailed schedule identifies the quantities of each raw material and component items. It also tells when items must be ordered and delivered so as to meet the master schedule for the final products.

- The purpose of M.R.P is to ensure that materials and components are available in the right quantities and at the right time so that finished products can be completed according to the master production schedule.



- The three inputs to the MRP are:

1. The master production schedule and other order data.
2. Bill of material file, which defines the product structure.
3. The inventory record file.

* Stores function:

- Stores play a vital role in the operations in any industry.
- The most important purpose served by the stores is to provide uninterrupted service divisions.

The functions of stores can be classified as follows:

1. To receive materials, components, tools, equipments and

other items and account for them.

2. To provide adequate and proper storage and preservation to various items.
3. To meet the demands of the consuming departments by proper issues and account for the consumption.
4. To minimize obsolescence, surplus and scrap through proper codification, preservation and handling.
5. To highlight stock accumulation, discrepancies and abnormal consumption and effect control measures. etc.

One bin system :

- A one bin inventory system is ~~simply a~~ a simple inventory control system which depends on replenishing supply at fixed time intervals and not at minimum stock level. These time intervals must be set in accordance to bin size, demand and lead time for the bin to be replenished up to a maximum limit no matter the rate of consumption.
- The one bin inventory system is useful when demand is predictable, stable and the process does not ~~demand~~ require a substantial amount of safety stock.
- eg — supply of sand, supply of ink to newspaper printing press etc.

Two bin system.

Two - bin inventory control is a system used to determine when items or materials used in production should be replenished. when items in the first bin have been depleted, an order is placed to refill or replace them.

The second bin is then supposed to have enough items to last untill the order for the first bin arrives.

- Two bin inventory control is almost always used for small and low value items that can be easily purchased and stored in bulk.

Material issue request

- Materials are issued only on the receipt of a properly authorized withdrawal form, which is usually called material issue request.
- A sample of material issue request —

Book No. ----- Indent No. ---
Shop/Deptt. --- Date ---

To

The storekeeper

Please issue the under-mentioned goods for job No......

S.No.	Material with specification	Quantity		Store ledger page No. where posted	Initials of storekeeper	Rate	cost	remarks	
		Demanded	Supplied						
	No. of items								

Received....... ......indentor

Bin Card

This is a card which is attached to each bin, rack, shelf or other container for stores. A record of all materials entering or leaving the bin and balance of material in hand is kept in this card. These cards are entered by the storekeeper and only the quantities are recorded.

Bin No......
Article.....

Minimum quantity
ordering level
Maximum quantity

Date	Quantity received	Quantity issued	Balance	Date	Quantity received	Quantity supplied	Balance

checked initials ---

checked initials----